Relations (3.10) and (3.12) together form a complete system of equations and boundary conditions for determining the motion of a body with a crack.

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THE SELFSIMILAR DYNAMIC PROBLEM OF A HYDRAULIC CRACK WHEN ITS SIDES INTERACT WITH A CLEAVING GAS FLOW*

YU.N. GORDEYEV

A selfsimilar solution of the problem of the propagation of a hydraulic crack, taking into account the interaction of its edges with a cleaving gas flow, is obtained. The influence of this interaction on the stress intensity factor (SIF) and the dynamic flow characteristics is studied.

In the problems of cleavage of an elastic half-space by a rigid wedge, one of the factors influencing the SIF is the force of friction between the wedge and elastic medium /1/. When solving the quasistationary problems of the hydrofracture of a stratum, the firctional forces arising between the gas flow and the crack edges are taken into account only in the equation of motion of the flow. The shear stresses connected with the frictional forces are neglected when the equations of the theory of elasticity are solved /2/. The selfsimilar dynamic problems of the propagation of cracks cleaved by a gas flow were studied in this approximation in /5, 6/, using the method of functionally invariant solutions /3, 4/. When the cracks are cleaved by means of compressed gas at high velocities, as happens in the case of impulsive hydrofracture /7/ and in the problems of explosive fracture /8/, the shear stresses arising at the crack edges can become considerable.

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1. Formulation of the problem. In the case of impulsive hydrofracture both liquid and compressed gas can be used as the working medium /7/. In the present paper only the cleavage of a crack by a gas flow will be discussed.

In order to describe the motion of the gas flow in a plane crack situated in the plane $x_2 = 0$, we shall use the laws of conservation of mass and momentum

$$\frac{\partial}{\partial t}(w\rho) + \frac{\partial}{\partial x}(\rho w u) = 0, \quad P = c^2\rho \tag{1.1}$$

$$\rho\left(\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u\right) + \frac{\partial}{\partial x}P = -F, \quad F = \lambda_s\rho\frac{u|u|}{w} \tag{1.2}$$

Here P, ρ, u are the pressure, density and velocity of the gas, w is the opening of the crack, F is the force of friction between the gas flow and the crack walls (a quadratic dependence of the resistance on the velocity is used, which holds for high velocity flows at $\mathrm{Re} = \rho wu/\mu \gg 1$, and μ is the viscosity of the gas), λ_s is the coefficient of resistance, c is the isothermal speed of sound and $2l(t)$ is the crack length at the instant t.

We will write the equations of motion of the elastic medium and Hooke's law in the form

$$w_\alpha = u_\alpha + v_\alpha, \quad \Box_1 u_\alpha = 0, \quad \Box_2 v_\alpha = 0 \tag{1.3}$$

$$\frac{\partial}{\partial x_2}u_1 = \frac{\partial}{\partial x_1}u_2, \quad \frac{\partial}{\partial x_1}v_1 = -\frac{\partial}{\partial x_2}v_2 \quad \left(\Box_\alpha = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{1}{c_\alpha^2}\frac{\partial^2}{\partial t^2}\right)$$

$$\sigma_{\alpha\beta} = \mu\left[\delta_{\alpha\beta}(\varkappa^2 - 2)\operatorname{div}\mathbf{w} + \frac{\partial}{\partial x_\beta}w_\alpha + \frac{\partial}{\partial x_\alpha}w_\beta\right]; \quad \alpha, \beta = 1, 2 \tag{1.4}$$

where $u_\alpha(x_1, x_2, t)$, $v_\alpha(x_1, x_2, t)$ are the potential and solenoidal components of displacement vector $\mathbf{w}(x_1, x_2, t)$; $\varkappa = c_1/c_2$; c_1, c_2 are the velocities of the longitudinal and transverse waves $(c_1 > c_2)$; $\sigma_{\alpha\beta}$ is the stress change tensor $\sigma_{\alpha\beta} = \overset{1}{\sigma}_{\alpha\beta} - \overset{0}{\sigma}_{\alpha\beta}$ and $\overset{0}{\sigma}_{\alpha\beta}$ is the initial-stress tensor /9/.

We will write the boundary conditions for Eqs.(1.3) and (1.4) in the form (v is the crack propagation velocity and σ is the homogeneous compressive stress)

$$\sigma_{22} = \sigma - P(x_1, t), \ \sigma_{12} = \tau = wF/2; \ x_2 = 0, \ |x_1| \leqslant vt \tag{1.5}$$
$$w_2 = 0, \ \sigma_{12} = 0; \ x_2 = 0, \ vt < |x_1| < c_1 t$$

We assume that the stress tensor component σ_{22} has the following root singularity at the crack ends:

$$\sigma_{22}(x_1, x_2 = 0, t) \xrightarrow[x_1 \to \pm vt]{} \frac{K_T(t)}{\sqrt{2\pi(|x_1| - vt)}} \tag{1.6}$$

Here $K_I(t)$ is the SIF.

The boundary conditions for Eqs.(1.1) and (1.2) describing the cleaving gas flow within the crack, have the form

$$P(x_1 = 0, t) = P_*, \ u(x_1 = 0 \pm 0, t) = \pm u_* \tag{1.7}$$

(P_*, u_* are constants). In order to describe the subsonic gas flow $(u_* < c)$ within the crack, it is sufficient to have a single boundary conditions, e.g. the first condition of (1.7), while in the case of a supersonic flow $(u_* > c)$ both conditions of (1.7) are necessary /10/.

2. Method of solving the elastic problem. In the case of a selfsimilar loading (P_0, τ_0 are constants with the dimensions of stress, t_0 is a constant with the dimensions of time, and l is a non-negative integer)

$$P(x_1, t) = P_0 \cdot (t/t_0)^{l-1} P(\xi); \ \tau(x_1, t) = \tau_0 \cdot (t/t_0)^{l-1}\tau(\xi); \ \xi = x_1/(vt) \tag{2.1}$$

we shall seek the solution of the boundary-value problem (1.3)-(1.6) using the method of the functionally invariant Smirnov-Sobolev solutions /3, 4/.

Let us introduce the functions homogeneous in the variables x_1, x_2, t,

$$U_\alpha = \partial^l u_\alpha/\partial t^l, \quad V_\alpha = \partial^l v_\alpha/\partial t^l; \quad \alpha = 1, 2 \tag{2.2}$$

(the quantities U_α satisfy the wave equation for the longitudinal waves, and V_α for the transverse waves).

Representing U_α, V_α as the real parts of analytic functions of the complex variables z_1, z_2 and substituting them into (1.3) and (1.4), we obtain $(k = 1, 2)$

$$\frac{\partial^l \sigma_{12}}{\partial t^l} = \mu \operatorname{Re}\left\{2 (U_2{}^l)' \frac{\partial z_1}{\partial x_1} + (1-\omega_2{}^2)(V_2{}^l)' \frac{\partial z_2}{\partial x_1}\right\} \tag{2.3}$$

$$\frac{\partial^l \sigma_{11}}{\partial t^l} = \mu \operatorname{Re}\left\{[\varkappa^2 (1+\omega_1^{-2}) - 2](U_2{}^l)' \frac{\partial z_1}{\partial x_2} - 2 (V_2{}^l)' \frac{\partial z_2}{\partial x_2}\right\}$$

$$\frac{\partial^l \sigma_{22}}{\partial t^l} = \mu \operatorname{Re}\left\{[\varkappa^2 (1+\omega_1^{-2}) - 2\omega_1^{-2}](U_2{}^l)' \frac{\partial z_1}{\partial x_2} + 2 (V_2{}^l)' \frac{\partial z_2}{\partial x_2}\right\}$$

$$\operatorname{Re} U_k{}^l (z_1) = U_k\left(\frac{x_1}{c_1 t}, \frac{x_2}{c_1 t}\right), \quad \operatorname{Re} V_k{}^l (z_2) = V_k\left(\frac{x_1}{c_1 t}, \frac{x_2}{c_1 t}\right)$$

$$z_k^{-1} = \frac{1}{c_k} \operatorname{ch}\left\{\operatorname{arch} \frac{1}{\xi_k} - i\varphi\right\}, \quad \xi_k = \frac{r}{c_k t}$$

$$r = (x_1{}^2 + x_2{}^2)^{1/2}, \quad \varphi = \operatorname{arctg}\left(\frac{x_2}{x_1}\right)$$

$$\frac{\partial}{\partial t} z_k = -z_k \frac{\partial}{\partial x_1} z_k = -\frac{1}{\sqrt{c_2^{-2} - z_k^{-2}}} \frac{\partial z_k}{\partial x_2} = \frac{z_k^2 \sqrt{c_k^{-2} - z_k^{-2}}}{x_2 z_k^{-1} - x_1 \sqrt{c_k^{-2} - z_k^{-2}}}$$

$$(U_1{}^l)' = \omega_1{}^{-1} (U_2{}^l)', \quad (V_1{}^l)' = -\omega_2 (V_2{}^l)', \quad \omega_k = z_k \sqrt{c_k{}^{-2} - z_k{}^{-2}}$$

The representation (2.3) for the analytic functions $U_k{}^l$, $V_k{}^l$ is identical with the one generally used /3/, provided that we replace z_k by $1/z_k$.

Now we replace the analytic functions $U_2{}^l$ and $V_2{}^l$ by the functions W and G, connected with them when $x_2 = 0$ $(z_1 = z_2 = z)$ by the relations

$$dU_2{}^l/dz = G'(z) + [1 - 2 (c_2/z)^2] W' \tag{2.4}$$
$$dV_2{}^l/dz = 2 (c_2/z)^2 W'$$

Substituting $(V_2{}^l)'$, $(U_2{}^l)'$ into (2.3), we obtain

$$\frac{\partial^l \sigma_{12}}{\partial t^l} = \frac{2\mu}{t} \operatorname{Re} G', \quad \frac{\partial^l \sigma_{22}}{\partial t^l} = -\frac{\mu z R(z)}{t c_2^{-2} \sqrt{z^{-2} - c_1^{-2}}} \operatorname{Im}\left\{W' + \frac{c_2^{-2}(c_2^{-2} - 2z^{-2})}{R(z)} G'\right\}, \tag{2.5}$$

$$\frac{\partial^l w_2}{\partial t^l} = \operatorname{Re}\{W + G\} \quad R(z) = (c_2^{-2} - 2z^{-2})^2 - 4z^{-2} \sqrt{z^{-2} - c_1^{-2}} \sqrt{z^{-2} - c_2^{-2}}$$

Let us formulate a boundary-value problem for the analytic functions W' and G' in the plane $z = 0$, taking into account (2.5) $(\operatorname{Im} z = 0 \pm 0)$

$$v < |z| < c_1, \ \operatorname{Re}'^{\pm} = 0, \ \operatorname{Re} W'^{\pm} = 0 \tag{2.6}$$
$$|z| < v, \ \operatorname{Re} G'^{\pm} = {}^1/_2 \mu^{-1} t \partial^l \tau / \partial t^l$$
$$\operatorname{Im}\{W' + c_2{}^{-2} (c_2{}^{-2} - 2z^{-2}) R^{-1}(z) G'\}^{\pm} = \pm t\mu^{-1} c_2{}^{-2} \sqrt{z^{-2} - c_1{}^{-2}} R^{-1}(z) z^{-1} \partial^l P / \partial t^l$$

Usually, when solving the selfsimilar plane dynamic problems of the propagation of a crack, we pose the boundary condition by stating that the shear stresses at the crack edges are equal to zero /3, 4/. In this case the general solution of the boundary-value problem of a crack with normal separation, can be expressed in terms of a single analytic function. In the case of the problem of the cleavage of a crack by a flow of gas where the crack interacts with the fluid at its edges, both the normal and shear stresses must be specified simultaneously, and the general solution of the boundary-value problem (2.6) cannot be expresed in terms of a single analytic function. If the shear stresses at the crack edges are small, (for example, when the rate of flow of the cleaving gas is small) $\tau(x_1, x_2 = 0, t) \approx 0$, $|x_1| < vt$, then $G \approx 0$ and the boundary-value problem (2.6) will be reduced to the usual boundary-value problem containing a single analytic function W only.

To solve the mixed boundary-value problem (2.6) we use the Keldysh-Sedov formulas

$$G' = \frac{t}{2\pi\mu i} \int_{-v}^{v} ds \frac{\partial^l \tau / \partial t^l}{s - z} + \frac{1}{(z^2 - v^2)^{l+1/2}} \sum_{j=0}^{2l-1} B_j z^j \tag{2.7}$$

$$W' = -\frac{c_2^{-2}(c_2^{-2} - 2z^{-2})}{R(z)} G' + \frac{1}{(z^2 - v^2)^{l+1/2}} \times$$
$$\left\{\frac{t c_2^{-2}}{\pi\mu} \int_{-v}^{v} ds \frac{(s^2 - v^2)^{l+1/2} \sqrt{s^{-2} - c_1^{-2}}}{s R(s)(s - z)} \frac{\partial^l}{\partial t^l} P + i \sum_{j=0}^{2l-1} A_j z^j\right\}$$

The solution (2.7) contains $4l$ constants A_j, B_j $(j = 0, \ldots, 2l-1)$, whose values can be found from the following system of equations:

$$\frac{\partial^j}{\partial t^j}\sigma_{22}(\pm vt \mp 0, 0, t) = -\frac{\partial^j}{\partial t_j}P(\pm vt, t) \tag{2.8}$$
$$\frac{\partial^j}{\partial t^j}\sigma_{12}(\pm vt \mp 0, 0, t) = \frac{\partial^j}{\partial t^j}\tau(\pm vt, t)$$

When the problems are symmetrical about the axis $x_1 = 0$, the number of constants to be determined is reduced.

3. ***The selfsimilar problem.*** When the rate of propagation of the crack is constant, problem (1.1)-(1.7) has a selfsimilar solution $(l = 1)$. The pressure, gas velocity and opening of the crack are expressed in terms of dimensionless functions of the selfsimilar variable

$$P(x_1, t) = P_0 P_a(\xi),\ u(x_1, t) = vu_a(\xi),\ w(x_1, t) = w_0 tw(\xi) \tag{3.1}$$
$$w_0 = \lambda_s v\varepsilon/4,\ \varepsilon = (v/c)^2,\ N = \sigma/P_0,\ \xi = x_1/(vt)$$

Eqs.(1.1) and (1.2) and conditions (1.7) written in the selfsimilar variables have the form (from now on the subscript a on the selfsimilar variables will be omitted)

$$d\ln P/d\xi = -\varepsilon(u-\xi)du/d\xi - u^2/w \tag{3.2}$$
$$du/d\xi = \{(u-\xi)w^{-1}[u^2 - dw/d\xi] - 1\}[1 - \varepsilon(u-\xi)^2]^{-1}$$
$$P(\xi = 0) = P_*/P_0,\quad u(\xi = 0 \pm 0) = \pm u_*/v \tag{3.3}$$

The Hugoniot conditions for an isothermal gas can be written in selfsimilar variables in the form

$$[P(u-\xi)] = 0,\ [P + \varepsilon P(u-\xi)^2] = 0$$
$$([f] = f(\xi + 0) - f(\xi - 0))$$

The following symmetry conditions about the axis $\xi = 0$ hold for the gas pressure P within the crack, the shear stress at its edges $\tau = \lambda_s\rho u|u|/8$, and the opening of the crack:

$$P(-\xi) = P(\xi),\ \tau(-\xi) = -\tau(\xi),\ w(-\xi) = w(\xi) \tag{3.4}$$

From the symmetry conditions it follows that when $l = 1$, expressions (2.7) will contain only the constant A_1 $(A_0 = B_0 = B_1 = 0)$.

Taking (3.4) into account, we shall approximate the pressure P and tangential stresses τ by the finite sums (δ_0 is the Heaviside function)

$$P(x_1, t) \approx P_0 \sum_{j=1}^{M} P_j[\delta_0(\xi_j - \xi) - \delta_0(-\xi_j - \xi)] \tag{3.5}$$
$$\tau(x_1, t) \approx \lambda_s\varepsilon P_0/8 \sum_{j=1}^{M} \tau_j[\delta_0(\xi_j - \xi) - 2\delta_0(-\xi) + \delta_0(-\xi_j - \xi)]$$

The equations of the theory of elasticity are linear, and hence the opening of the crack and SIF can also be approximated by the finite sums

$$w(\xi) \approx \sum_{j=1}^{M}[P_j w^{1j}(\xi) + \tau_j w^{2j}(\xi)] \tag{3.6}$$
$$K_I(t) \approx \sum_{j=1}^{M}[P_j K_I^{1j}(t) + \tau_j K_I^{2j}(t)]$$

Let us find $w^{\alpha l}(\xi)$, $K_I{}^{\alpha l}(t)$ $(\alpha = 1, 2)$ for the special case of loading (3.5): $P_j = P_l\delta_{jl}$; $\tau_j = \tau_l\delta_{jl}$; $j = 0, \ldots, M$ (there is no summation over l).

Substituting the load (3.5) for $P_j = P_l\delta_{jl}$, $\tau_j = \tau_l\delta_{jl}$ into (2.7) $l = 1$, $A_0 = B_0 = B_1 = 0$ and integrating them over z and t we obtain, taking into account conditions (1.4)

$$w(\xi) = P_l w^{1l}(\xi) + \tau_l w^{2l}(\xi),\quad K_I(t) = P_l K_I{}^{1l}(t)\quad (K_I{}^{2l} \equiv 0) \tag{3.7}$$

$$w^{1l}(\xi) = \sigma_1 \sqrt{1-\xi^2} - \sigma_2 \varkappa(\xi) [\xi_l^2 \sqrt{1-\xi_l^2}]^{-1}$$

$$\varkappa(\xi) = |\xi| \ln \frac{|1-\zeta\xi_l/|\xi||}{1+\zeta\xi_l/|\xi|} - \xi_l \ln \frac{|1-\zeta|}{1+\zeta}, \quad \zeta = \sqrt{\frac{1-\xi^2}{1-\xi_l^2}}$$

$$w^{2l}(\xi) = \frac{P_0 v}{\pi\mu w_0} \frac{\alpha_s}{R_1(\xi_l)} [2(2-m^2\xi_l^2) - 4\sqrt{1-n^2\xi_l^2}\sqrt{1-m^2\xi_l^2}] \times$$
$$\times (\xi_l - |\xi|)\, \delta_0(\xi_l^2 - \xi^2)$$

$$K_I^{1l}(t) = -\sigma_1 R_1(1) \frac{\mu w_0}{c_1 n m^2} \sqrt{\frac{\pi v t}{1-n^2}}$$

$$\sigma_1 = \sigma_2 \frac{K(1-\xi_l^2) + 2\xi_l}{\xi_l^2}, \quad \sigma_2 = \frac{P_0 v}{\pi\mu w_0} \frac{m^2\xi_l^4}{R_1(\xi_l)} \sqrt{1-n^2\xi_l^2}\sqrt{1-m^2\xi_l^2}$$

$$n = v/c_1, \ m = v/c_2, \ \alpha_s = \pi\lambda_s\varepsilon/16, \ R_1(\xi) = \xi_l^4 v^4 R(\xi)$$

The constant K is given by the complete elliptic integrals /5/.

The dynamic selfsimilar problem of the propagation of a hydrofracture in an elastic space was solved in /5/ for the case when the frictional forces between the flow and the crack walls are neglected $\tau(x_1, t) = 0$ $(\tau_l = 0)$.

From (3.7) it follows that, just as in the static solution when the load satisfies the symmetry conditions (3.4), then the shear stress τ will not contribute directly to the SIF. However, since the opening of the crack depends on shear stresses at its edges τ, they will affect, through the equations of motion (3.2), the distribution of the gas pressure within the crack and hence the SIF.

The problem of the cleavage of a crack by as flow (3.2), (3.3) was solved numerically. The numerical method was based on the use of expansions (3.5), (3.6) /5/.

Figs.1-3 show some of the computational results with respect to the dimensionless parameters $\varepsilon = 2.25$, $n = 0.2$, $m = 0.3$ and $P_0/(\pi\mu\varepsilon\lambda_s) = 1$.

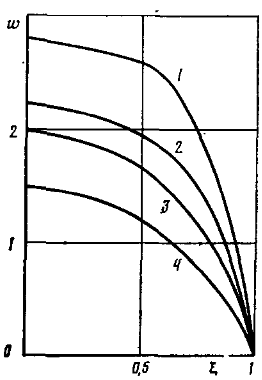

Fig.1

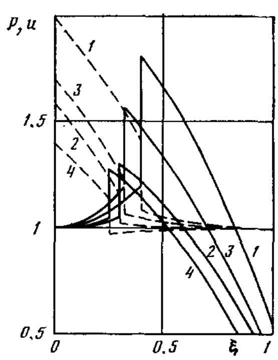

Fig.2

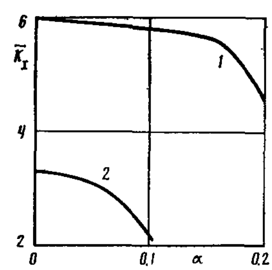

Fig.3

The opening of the crack w, pressure P and the rate of gas flow within the crack u are shown in Figs.1 and 2. Curves *1* and *2* correspond to the values $N = 0, \alpha_s = 0$ (1), $\alpha_s = 0.2$ (2), and curves *3* and *4* to $N = 0.3$, $\alpha_s = 0$ (3), $\alpha_s = 0.1$ (4).

The dimensionless parameter α_s, proportional to the square of the ratio of the crack propagation velocity and the speed of sound in the gas, and inversely proportional to the coefficient of resistance, characterizes the frictional forces between the cleaving gas flow and the crack edges. In the case of impulsive hydrofracture, when the crack is cleaved by a supersonic gas flow, we see from Figs.1 and 2 that the frictional forces begin to exert a significant influence on the crack profile w and on the distribution of gas pressure P in the flow. Moreover, then the gas enters the crack at supersonic velocities, a retardation shock wave forms within the flow /5/. Shear stresses at the crack edges increase as the parameter α increases, and the opening of the crack is reduced. The hydraulic resistance of the crack increases, and this leads to expulsion of the retardation shock wave from the crack and a more rapid drop in the gas pressure along the crack, and hence to decrease in the value of the SIF. Fig.3 shows the relation $\overline{K}_I = [K_I/(P_0\sqrt{\pi v t})] \times 10^2$ for $N = 0$ (curve *1*) and for $N = 0.3$ (curve *2*).

Thus, when the stratum is impulsively hydraulically cleaved and when the medium is fractured explosively, the frictional forces arising between the gas flow and the crack edges may exert a considerable influence on the dynamic of crack propagation.

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THE EQUATIONS OF MOTION OF CONDENSED MEDIA WITH CONTINUALLY KINETIC FRACTURE*

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A continual model of fracture /1, 2/ within the framework of which the degree of damage to the material is determined by the volume of the micropores or voids formed as a result of increasing tensile stresses, is reformulated to cover the case of viscoelastic media with finite deformations. As a result, equations of motion of a viscoelastic medium with continual fracture are proposed. In the case of a medium without fracture the equations are identical with the equations of motion /3-6/, and when the damage is small and the loading uniaxial, they reduce to the well-known equations /1/. The properties of certain simplest flows are studied using the model proposed.

A large volume of literature exists, dealing with the rheological models of a continuous condensed medium, describing strength effects. A phenomenological approach to constructing the defining relations, including, in the limit, the hydrodynamic as well as elastic modes of motion of the material, which retains its contnuity, is given in /3, 7/. The problem of including fracture in such models has received less attention. A survey is given in /4/ of work done up till now dealing with this problem, and a theory of the continual fracture of non-linearly elastic model based on a phenomenological approach is developed. A second rank tensor whose properties were studied in /4/ is used as the macroscopic measure of material damage. By virtue of the assumptions made in /4/, it is established that the increase in the damage in thermo-elastic media is governed not by the kinetic equation, but by a finite

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